Exam 2 - Notes

- **Cauchy-Euler Equations** - Have the form:

$$at^2y''(t) + bty'(t) + cy(t) = 0$$

with associated characteristic equation $ar^2 + (b-a)r + c = 0$.

1. Two distinct, real r's: general soln is $y(t) = c_1t^{r_1} + c_2t^{r_2}$
2. One real r: general soln is $y(t) = c_1t^r + c_2t^r \ln t$
3. Complex $r = \alpha \pm \beta i$: general soln is $y(t) = c_1t^\alpha \cos(\beta \ln t) + c_2t^\alpha \sin(\beta \ln t)$

- **Reduction of Order**

Equations of the form: $y''(t) + p(t)y'(t) + q(t)y(t) = 0$.

If one solution $y_1(t)$ to the DE is known; then let $y_2(t) = u(t)y_1(t)$.

- **Variation of Parameters**

$$y''(t) + p(t)y'(t) + q(t)y(t) = f(t)$$

$p(t)$ and $q(t)$ may be constants.
Look for a particular solution of the form: $y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t)$, where v_1 and v_2 satisfy the following conditions:

1. $v_1'y_1 + v_2'y_2 = 0$
2. $v_1'y_1' + v_2'y_2' = f$

- **Spring-mass system:** $my'' + by' + ky = 0$

you will often need Hooke's Law: $F = kL$.

$F_w = mg, \quad T = \dfrac{2\pi}{\omega}, \quad A = \sqrt{c_1^2 + c_2^2}, \quad \phi = \begin{cases} \tan^{-1}\left(\dfrac{c_1}{c_2}\right) & \text{if } c_1 > 0,\ c_2 > 0; \quad (Q_1, Q_4) \\ \pi + \tan^{-1}\left(\dfrac{c_1}{c_2}\right) & \text{if } c_2 < 0; \quad (Q_2, Q_3) \end{cases}$

- **Integration**

$$\int \frac{1}{a^2 + u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C \qquad \int \frac{2x-1}{x^2}e^{2x}dx = \frac{e^{2x}}{x} + C$$

$$\int \sec t\,dt = \ln|\sec t + \tan t| + C \qquad \int \csc t\,dt = \ln\left|\tan\left(\frac{t}{2}\right)\right| + C$$

- **Trig Identities**

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2} \qquad \sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$