Related Rates

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Calculus was invented to describe and predict phenomena of change (planetary motion, free fall, population dynamics, etc.). In many practical applications several related quantities vary together, as you would expect, the rates at which they vary are also related.

General Road Map

1. Draw a good (BIG) picture.
2. Label - introduce variables as necessary (consider what is a function of what).
   Write Down the given information.
   Write Down what you are looking for.
3. Identify the algebraic relationship between the variables and write down this equation.
   Occasionally you will need to eliminate a variable.

3a. You are NOT stuck until you have done 1,2, and 3.
   If you have, and now believe you are stuck, ... WRITE DOWN STUFF.

4. Differentiate both sides of your equation (the relationship you found in 3).
   You differentiate with respect to t (time).

   Do NOT introduce specific values for the rates or variables prior to differentiating.
   ALWAYS obtain a general formula that involves the rates of change at any arbitrary time t.

5. Now, after all of the above is completed, substitute the known values and given rates to find the unknown rate of change.

CLASS EXAMPLES

DAY ONE

1. (classic warm-up) Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast is the area of the spill increasing when the radius is 60 ft?

2. Kate, while driving to Meadville for break received a speeding ticket. The officer claimed the car was clocked at 105 ft/sec from a helicopter stationary at 1000 feet elevation, at the instant the car was 2000 feet from the helicopter (line of sight).
   Note: the posted speed limit was 75 mi/hr = 110 ft/sec (so you might claim "NOT GUILTY!").
   Innocent or Guilty ?

CLASS EXAMPLES

DAY TWO

1. (Note, for this warm-up example I have ignored units in favor of a simple problem.)
   An approximate relationship between the pressure, P and the volume, V of the vapor in a diesel engine cylinder is $PV^{1.4} = k$, where k is a constant. At the instant $P = 12, V = 2.8$, and the volume is increasing at a rate of $\frac{1}{2}$, at what rate is the pressure changing?

2. A water tank in the shape of an inverted circular cone with base radius $2m$ and height $4m$. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is $3m$ deep.