MATH 413
Fall 2008
Lab 1

Exercise 1 **(10 pt)**

This assignment is designed to compare different methods for the determination of roots. In particular, the goal is to find all points of the intersection of the two curves: $f_1(x) = x^2 + 2x + 3$ and $f_2(x) = -x + 2$ using the different methods listed below. In all cases use $\epsilon = 0.0001$, record the numerical values and the number of iterations.

- Graph the two curves to see where they intersect. For each point of the intersection α, choose initial values of x (x_{upper} and x_{lower}) that will enable the Bisection Method to approximate α.
- Use the Bisection Method to approximate α using the initial values from above.
- Generate a fixed point method and use the values x_{upper} and x_{lower} to approximate each α. Should both x_{upper} and x_{lower} fail to find the root, find a different starting values.
- Compare the methods with regards to efficiency, accuracy and with respect to whatever else you may find worth noticing.

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Lab 2
Continuation of Lab 1

Exercise 1 **(10 pt)**
Remember lab assignment 1.
This assignment is designed to compare different methods for the determination of roots. In particular, the goal is to find all points of intersection of the two curves: $f_1(x) = x^2 + 2x + 3$ and $f_2(x) = -x + 2$ using the different methods listed below. In all cases use $\epsilon = 0.0001$, record the numerical values and the number of iterations.

Also remember the information obtained from the graphs in Lab 1.
Graph the two curves to see where they intersect. For each point of intersection α, choose initial values of x (x_{upper} and x_{lower}) that will enable the Methods to approximate α.

- For each α use Newton's Method with both x_{upper} and x_{lower} as initial values in order to approximate α. Should both x_{upper} and x_{lower} fail to find the root, find different starting values.
- Sketch the graph of $f(x) = \exp(x - 2) - x + 1$.
- Verify that 2 is a root of multiplicity 2.
- Use Newton's Method with $\epsilon = 0.0001$ and $x = 0$ to approximate the root. Note the number of iterations. Verify experimentally that the root has multiplicity 2.
- Use the P-Modified Newton's Method with $\epsilon = 0.0001$ and $x = 0$ to approximate the root. Note the number of iterations.
- Compare the methods with regards to efficiency, accuracy and with respect to whatever else you may find worth noticing.

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Lab 3
Continuation of Lab 1

Exercise 1 **(10 pt)**
This assignment is designed to compare different methods for the for the determination of roots. In particular, the goal is to find all points of intersection of the two curves: $f_1(x) = x^2 + 2x + 3$ and $f_2(x) = -x + 2$ using the different methods listed below. In all cases, use $\epsilon = 0.00001$, record the numerical values and the number of iterations.

Programs for Fixed Point Method and Fixed Point Method with Aitken's Acceleration are available: pantherfile.uwm.edu/stevesch/public

- Use the Secant Method (SECANT22) to approximate each point of intersection. Use starting intervals [-1, 0] and [-3, -2].
- Use the Method of False Position (FALPOS23) to approximate each point of intersection. Use starting intervals [-1, 0] and [-3, -2].
- Compare the methods with regards to efficiency, accuracy and with respect to whatever else you may find worth noticing.

Consider the function $f(x) = e^{x-2} - x + 1$.

- Find a fixed point method to approximate the root of f.
- Find a fixed point method with Aitken's Acceleration to approximate the root.
- Compare the methods with regards to efficiency, accuracy and with respect to whatever else you may find worth noticing.

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Lab 4

Exercise 1 **(10 pt)**

Given $A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix}$ and $b = \begin{bmatrix} \frac{25}{12} \\ \frac{77}{60} \\ \frac{19}{20} \\ \frac{319}{420} \end{bmatrix}$.

- Verify that $x = (1, 1, 1, 1)$ is the true solution to $Ax = b$.
- Create new matrices A' and b' by rounding all entries of A and b to 4 significant digits. Determine the actual solution x to the equation $A'x = b'$.
- Use Gaussian elimination (GAUMPP62) to solve the system $A'x = b'$. Find the relative error by computing $\frac{\|x-a\|}{\|x\|}$ where a is the approximate solution and x is the true solution.
- Use 0.24 as your value for $A'[4, 1]$. Run Gaussian elimination again and find the relative error. Compare this with b).
- Use 2.08 as your value for $b'[1]$. Find the relative error after running Gaussian elimination and compare with the previous results.
- Loosely speaking, a system is said to be stable if a small change in the initial value results in at most a small change in the result. What can you say about the stability of this system?

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Lab 5

Exercise 1 **(10 pt)**

Given $A = \begin{bmatrix} 1.19 & 2.11 & -100 & 1 \\ 14.2 & -0.122 & 12.2 & -1 \\ 0 & 100 & -99.9 & 1 \\ 15.3 & 0.11 & -13.1 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} 1.12 \\ 3.44 \\ 2.15 \\ 4.16 \end{bmatrix}$.

- Run Gaussian Elimination (GAUSEL61) to verify that the solution is approximately $x_1 = 0.17682530, x_2 = 0.01269269, x_3 = -0.02065405, x_4 = -1.18260870$.
- Use Gaussian Elimination with Partial Pivoting (GAUMPP62) with (Digits : = 5) to solve this system.
- Use Gaussian Elimination with Refinement (ITREFN74) with (Digits : = 5) to solve this system.
- Multiply row 1 of matrix A and vector b by 100, and divide row 2 of matrix A and vector b by 100.
- Use Gaussian Elimination with Partial Pivoting (GAUMPP62) with (Digits : = 5) to solve this system.
- Use Gaussian Elimination with Partial Pivoting and Scaling (GAUSPP63) with (Digits : = 5) to solve this system.
- Use Gaussian Elimination with Refinement (ITREFN74) with (Digits : = 5) to solve this system.
- Comment on your results with regard to the accuracy of each method. Specifically, address any changes resulting from multiply a row by a scalar.

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Lab 6

Exercise 1 **(10 pt)**

Solving Linear Systems by Jacobi Iteration Method

$$(A)\left\{\begin{array}{rrrrl} 5x_1 & -x_2 & +2x_3 & -x_4 & =3 \\ x_1 & +4x_2 & -x_3 & +x_4 & =2 \\ x_1 & -x_2 & -4x_3 & +x_4 & =5 \\ & x_2 & & -3x_4 & =0 \end{array}\right\} \text{ and } (B)\left\{\begin{array}{rrrrl} 2x_1 & -x_2 & +2x_3 & -x_4 & =0 \\ x_1 & +3x_2 & -x_3 & +x_4 & =2 \\ x_1 & -2x_2 & +x_3 & +x_4 & =0 \\ & x_2 & +x_3 & -2x_4 & =-1 \end{array}\right\}$$

- Check that the exact solution is $(1, 0, -1, 0)$ for both systems.
- Determine whether each system is diagonally dominant.
- When we solve the system (A) by Jacobi Iteration with certain initial guess and tolerance $\epsilon = 0.00001$, can we be sure it will converge to the exact solution? Why or why not?
- Repeat for the system (B).
- Solve the system (A) by Jacobi Iteration with the initial guess $(0, 0, 0, 0)$ first and then $(1, 2, 3, 4)$.
- Repeat for the system (B).
- Try to solve the following systems by the Jacobi method with $\epsilon = 0.00001$.

$$(C)\left\{\begin{array}{rrrl} x_1 & +2x_2 & -2x_3 & =1 \\ x_1 & +x_2 & +x_3 & =3 \\ 2x_1 & +2x_2 & +x_3 & =5 \end{array}\right\} \text{ and } (D)\left\{\begin{array}{rrrl} 5x_1 & +3x_2 & +4x_3 & =12 \\ 3x_1 & +6x_2 & +4x_3 & =13 \\ 4x_1 & +4x_2 & +5x_3 & =13 \end{array}\right\}$$

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Lab 7

Solving Linear Systems by iterative methods
We want to solve the following two linear systems by iterative methods.
(We can check the exact solution is $(1, 0, -1, 0)$ for both systems.)

$$(A) \left\{ \begin{array}{rrrrl} 5x_1 & -x_2 & +2x_3 & -x_4 & = 3 \\ x_1 & +4x_2 & -x_3 & +x_4 & = 2 \\ x_1 & -x_2 & -4x_3 & +x_4 & = 5 \\ & x_2 & & -3x_4 & = 0 \end{array} \right\} \text{ and (B) } \left\{ \begin{array}{rrrrl} 2x_1 & -x_2 & +2x_3 & -x_4 & = 0 \\ x_1 & +3x_2 & -x_3 & +x_4 & = 2 \\ x_1 & -2x_2 & +x_3 & +x_4 & = 0 \\ & x_2 & +x_3 & -2x_4 & = -1 \end{array} \right\}$$

Exercise 1 **(10 pt)**
Gauss-Seidel Iteration and SOR

- Solve the system (A) by Gauss-Seidel Iteration (GSEITR72) with the initial guess $(0, 0, 0, 0)$ and $(1, 2, 3, 4)$. Note the speed of convergence. Use tolerance $\epsilon = 0.00001$.
- Repeat this for the system (B).
- Why does Gauss-Seidel Iteration converge faster than Jacobi Iteration?
- Solve the system (A) by SOR (SORITR73) with the initial guess $(0, 0, 0, 0)$ and $(1, 2, 3, 4)$ and for varies relaxation factors. Note the speed of convergence. Use tolerance $\epsilon = 0.00001$.
- Repeat this for the system (B).
- Compare Jacobi (from last week's lab), Gauss-Seidel, and SOR.

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Lab 8

Exercise 1 **(10 pt)**

Newton's Method and Generalized Newton's Method for Non-Linear Systems.

Consider the following system of equations:
(1) $xyz - x^2 + y^2 = 1.34$
(2) $xy - z^2 = 0.09$
(3) $e^x - e^y + z = 0.41$

- Use Newton's Method with $\epsilon = 0.00001$ to find the solution using an initial approximation of $(1, 1, 1)$.
- Try the Generalized Newton's Method with various ω's. Note the results.
- Reorder the system to (3), (1), (2) and try the Generalized Newton's Method again.
- Compare the resuls of parts 2 and 3, and note the differences.
- Compare the methods with regards to efficiency, accuracy and with respect to whatever else you may find worth noticing.

Consider the system:
$y - \tan(x) = 21$
$5x + \tan(y) = 30$

- Graph the system to find the approximate solution:
  `plot([tan(x)+21,arctan(30-5*x)],x=0..10,y=-5..5);`
- Try to find the root near (8, -1.25) with both Newton's Method and Generalized Newton's Method. Try various ω's in Generalized Newton's Method.
- Compare both methods.

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Lab 9

This lab requires the programs **lagrange** and **Newton Fore Diff**, both found at https://pantherfile.uwm.edu/stevesch/public/.

1. Use the table provided to find the value of $f(x)$ at $x = 2.8$ by the Lagrange's method of interpolation.

x	.3	1.3	2.3	3.3	4.3	5.3
$f(x)$	.2955202	.9635581	.7457053	$-.1577457$	$-.9161661$	$-.8322674$

a. Use the 1st, 2nd, 3rd, 4th and 5th order interpolating polynomial (order data correctly) to interpolate for $f(2.8)$ and to find the absolute error. (Note : the exact value is $\sin(2.8) = .3349882$.)
b. Graph the polynomials of order 2, and 5, compare them to $y = \sin(x)$, which is the exact curve.

2. M. S. Selim and R. C. Seagraves studied the kinetics of elution of copper compounds from ion-exchange resins. The normality of the leaching liquid was the most important factor in determining the diffusivity. Their data were obtained at convenient values of normality; we desire a table of D for integer values of normality ($N = 0.0, 1.0, 2.0, 3, 0, 4.0.5.0$).

N	Dx10^6, cm^2/sec	N	Dx10^6, cm^2/sec
0.0521	1.65	0.9863	3.13
0.1028	2.10	1.9739	3.06
0.2036	2.27	2.443	2.92
0.4946	2.76	5.06	2.07

a. Using interpolation up to the highest possible order, find interpolation values for $N = 0$ through $N = 5$.

b. Compare the results and pick the best interpolation value for $N = 0$ through $N = 5$.

c. Which values yields the worst results? What may be the reason for it?

3. Redo problem 2.a and 2.b using Newton's Forward Differences. Try to find the best choice for both the degree of the polynomial and diagonal. Remember to reorder the data points as needed.

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Lab 10

Least Squares

1. Given the data

x	0	.25	.66666667	.75	1	1.25
$f(x)$	$-.75$	1.80	3.00	2.90	1.07	$-.8$

a. Use the Least Squares program to find a second degree polynomial approximation and a fifth degree polynomial approximation over the interval $[0, \frac{5}{4}]$.

b. Use the same routine to find an approximate curve of the form $f(x) = a\sin(x) + b\cos(x)$.

d. If you had to extrapolate at $x = 2$, which of these curves would you use and why?

Differentiation

1. a. Graph the function $f(x) = \ln(x)$ over $[90, 110]$. Set the axis corners to $(90, 0)$ and $(110, 20)$.

```
plot(ln(x),x=90..110,y=0..20);
```

b. Use the Numerical Differentiation program to approximate the derivative at $x = 100$. Use $h = 0.5$. For each of the first three columns, find the best approximation (true value is .01). Which gives the best approximation? Does this always occur for each method at the smallest value of h? Why do you think this happens?

2. a. Using $f(x) = x\cos(2x)$ approximate the first and second derivatives at $x = \frac{5}{2}$. (Let $h = 0.5$).

b. Which of the methods gives the best approximation to the first derivative?

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Lab 11

Exercise 1 **(10 pt)**

Use the programs **trap** available at http://pantherfile.uwm.edu/stevesch/public/ and **csimpr41** on the cd.

- Find the actual value of the integral $\int_3^4 (\frac{3}{2}\sqrt{x} + 3x^2)dx$.
- Use the Trapezoidal Rule to evaluate the integral using $n = 2, 4, 8, 20$ subdivisions. Find the absolute error for each approximation.
- Use Simpson's Rule to evaluate the integral using $n = 2, 4, 8, 20$ subdivisions. Find the absolute error for each approximation.
- Compare your results for each method.

Recall from calculus that the proper method of evaluating the integral $\int_a^\infty f(x)dx$ is to evaluate $\lim_{b\to\infty} \int_a^b f(x)dx$. A numerical approach is to make the substitution $t = \frac{1}{x}$, so $dx = -\frac{1}{t^2}dt$, and evaluate $\int_0^{\frac{1}{a}} \frac{1}{t^2} f(\frac{1}{t})dt$.

- Show that
$$\int_0^\infty xe^{-x}dx = \int_0^1 xe^{-x}dx + \int_0^1 \frac{e^{-\frac{1}{x}}}{x^3}dx$$
Find the actual value of this integral, as well as the integral $\int_0^1 xe^{-x}dx$.
- Use Simpson's Rule with $n = 40$ to approximate $\int_0^1 \frac{e^{-\frac{1}{x}}}{x^3}dx$ by choosing various lower bounds a for the integral $\int_a^1 \frac{e^{-\frac{1}{x}}}{x^3}dx$.
- Approximate the integral $\int_0^\infty xe^{-x}dx$ using your results. Find the absolute error for several different values of a.
- Which value of a yielded the best results?

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Lab 12

Use the programs **Gauss Quad Legendre**, **Gauss Quad Laguerre**, and **EULER** available at http://pantherfile.uwm.edu/stevesch/public/.

Recall that $\int_0^\infty xe^{-x}dx = 1$.

- Use Gaussian - Legendre quadrature to approximate the integral with 2 and 6 points.
- Use Gaussian - Laguerre quadrature to approximate the integral with 2 points.
- Find the absolute error for each approximation. Compare your answers using each method and with Simpson's method from the last lab.
- Solve using Euler's Method: $y' = x^2(\sqrt{y} + x)$, $y(1) = 2$, for $x \in [1, 2]$ using $h = 0.1$ and $h = 0.05$.