MATH 280 Modern Algebra
Fall 2015 · Syllabus

Class Information
Instructor: Dr. Lauren Williams
Class Meeting: MWF 9:15 - 10:20 in Hirt 209
Office: Old Main 401 (Tower)
Office Phone: (814) 824-2226
Office Hours: Mon 10:30-12, Tues 12-1, Wed 2:15-3:15, Thurs 11-12, Fri 10:30-12
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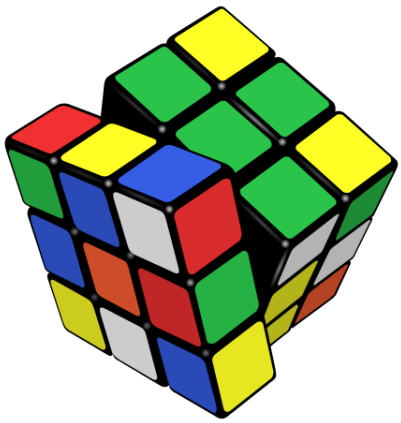
Course Description
This is the first semester of a year long sequence on the study of algebraic structures. Course topics include the properties of numbers, equivalence relations, groups, rings, fields, direct products, homomorphisms and isomorphisms, and the natural development of various number systems.

Course Objectives
On successful completion of the course, students should be able to:

- provide the definitions of algebraic objects, and know some examples of each.
- develop abstract and critical reasoning by studying and writing mathematical proofs.
- understand the connection between modern algebra and other branches of mathematics.
- relate the material learned in this course to prerequisite courses.
- recognize algebraic structures and objects in everyday situations.
- learn about the historical development of modern algebra.

Textbook
Contemporary Abstract Algebra, by Joseph Gallian, 8th Edition (older editions are fine too). No other supplies are required for the course.



Depiction of a non-abelian group of order 43,252,003,274,489,856,000

Lectures and Notes

Partial notes for each lecture, including definitions and theorem statements, will be available on the course website. You should read the relevant course notes for each topic before class, and you're encouraged to come to class with questions. By seeing the definitions and theorems ahead of time, you'll be able to focus on understanding them in class, rather than concentrating on writing.

The notes you'll be given are not complete, but will complement the examples and proofs of important theorems we'll cover in class. How you choose to use these notes is up to you. One suggestion for studying is to recopy all notes (handouts and those you take in class) in a way that's clear to *you*. As you're doing so, you're likely to uncover questions you hadn't thought of before.

If you miss a class, make sure you get the additional notes from a classmate. Attendance is not required nor part of your grade, but coming to class regularly is in your best interest.

Homework

You will have several assignment due throughout the semester (generally every Friday). You should expect to spend a fair amount of time on each assignment - don't wait until the night before it's due to get started! You are free to work together on your assignments, but everyone must submit their own work, in their own words. The lowest homework grade will be dropped. Late homework will not be accepted, unless you talk to me before it's due.

Exams

We will have a midterm exam and a final exam. The midterm exam will have a take home portion, followed by an in class portion. Unlike the homework assignments, you are not permitted to work together on the take home portion of the midterm exam. The in class portion will be comprised of shorter questions based on definitions, while the take home will include some proofs. The final exam will be a cumulative exam with no take home portion.

Exam Dates:

Midterm Exam (In Class and Take Home Portion Due): Monday, October 19

Final Exam: Friday, December 11, 8:00-10:00 am

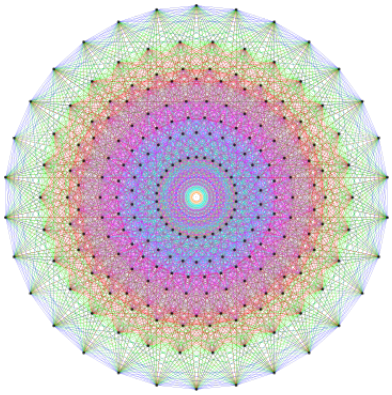
Final Grades

Grades will be calculated as follows:

60% - Average of homework assignments (lowest grade dropped)
20% - Midterm Exam
20% - Final Exam

Grading scale:

F	D	D+	C	C+	B	B+	A
0-59	60-64	65-69	70-77	78-83	84-89	90-93	94-100



Mapping of the exceptional simple Lie group E8

Additional Resources on the Course Website

The site for this course contains lots of useful material and information, including:

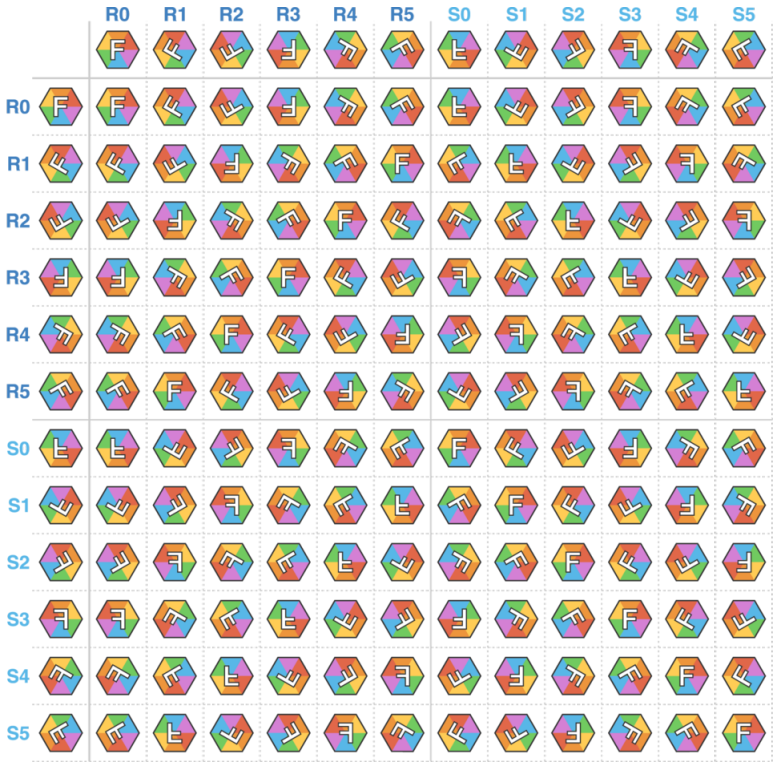
- all information found in this syllabus
- a detailed (and updated) course schedule
- lecture notes for each class meeting, posted at least one week before the meeting
- copies of any course handouts
- applets designed for our course that help reinforce definitions and concepts
- a customized study guide, with all definitions and theorems, outlines of proofs, and many practice questions with solutions
- links to additional resources on the web, such as free algebra textbooks and study materials
- suggestions and tutorials for software related to modern algebra
- a calculator to determine what grade you'll need on the final exam to earn your desired final grade, based on your homework and midterm grades

Support of the Mercy Mission

This course supports the mission of Mercyhurst University by creating students who are intellectually creative. Students will foster this creativity by: applying critical thinking and qualitative reasoning techniques to new disciplines; developing, analyzing, and synthesizing scientific ideas; and engaging in innovative problem solving strategies.

Learning Differences

In keeping with college policy, any student with a disability who needs academic accommodations must call Learning Differences Program secretary at 824-3017, to arrange a confidential appointment with the director of the Learning Differences Program during the first week of classes.



Cayley table for dihedral group of order 12

Math 280 Modern Algebra Course Schedule - Fall 2015

Aug 26	**Topic**: Course Intro, Sets	
Aug 28	**Topic**: Properties of Numbers, Modular Arithmetic	
Aug 31	**Topic**: Logic, Proofs	
Sept 2	**Topic**: Linear Algebra	
Sept 4	**Topic**: Functions, Equivalence Relations	**Assignment 1 Due**
Sept 7	*Labor Day - NO CLASS*	
Sept 9	**Topic**: Binary Operations, Groups	
Sept 11	**Topic**: Groups, Properties of Groups	**Assignment 2 Due**
Sept 14	**Topic**: Properties of Groups, Order	
Sept 16	**Topic**: Subgroups	
Sept 18	**Topic**: The Dihedral Group	**Assignment 3 Due**
Sept 21	**Topic**: Cayley Tables	
Sept 23	**Topic**: Centers and Centralizers	
Sept 25	**Topic**: Cyclic Groups	**Assignment 4 Due**
Sept 28	**Topic**: Cyclic Groups	
Sept 30	**Topic**: Permutations	
Oct 2	**Topic**: The Symmetric and Alternating Groups	**Assignment 5 Due**
Oct 5	**Topic**: The 15 Puzzle and The Rubix Cube Group	
Oct 7	**Topic**: Group Homomorphisms	
Oct 9	**Topic**: Group Homomorphisms	**Assignment 6 Due**
Oct 12	**Topic**: Group Isomorphisms	
Oct 14	**Topic**: Group Isomorphisms	**Assignment 7 Due**
Oct 16	*Mid-Semester Break - NO CLASS*	
Oct 19	**Midterm Exam**	**Take Home Portion Due**
Oct 21	**Topic**: Cosets	
Oct 23	**Topic**: Lagrange's Theorem	
Oct 26	**Topic**: External Direct Products	
Oct 28	**Topic**: Normal Subgroups	
Oct 30	**Topic**: Factor Groups	**Assignment 8 Due**
Nov 2	**Topic**: Factor Groups	
Nov 4	**Topic**: The Fundamental Theorem of Finite Abelian Groups	
Nov 6	**Topic**: Rings	**Assignment 9 Due**
Nov 9	**Topic**: Rings	
Nov 11	**Topic**: Integral Domains	
Nov 13	**Topic**: Integral Domains	**Assignment 10 Due**
Nov 16	**Topic**: Ideals	
Nov 18	**Topic**: Factor Rings	
Nov 20	**Topic**: Ring Homomorphisms	**Assignment 11 Due**
Nov 23 - 27	*Thanksgiving Break - NO CLASS*	
Nov 30	**Topic**: Polynomial Rings	
Dec 2	**Topic**: Divisibility in Integral Domains	
Dec 4	Final Exam Review	**Assignment 12 Due**
Dec 7	*Reading Day - NO CLASS*	
Dec 11	**FINAL EXAM 8:00-10:00**	

Why Study Modern Algebra?

The following is an excerpt from Joseph Gallian's website on why modern algebra (also known as abstract algebra) is a valuable subject for math and math ed majors:

1. Even though many students take a course is discrete math where they study various proof techniques many of them seem not to absorb this material well. Abstract algebra provides them much more practice at this in a different context than discrete math does.
2. High school math teachers should be very adept at modular arithmetic. Cyclic groups is where they learn this well.
3. Group theory is the mathematics of symmetry–a fundamental notion in science, math and engineering. For example, the symmetry group of a molecule reveals some of its possible (or impossible) chemical properties.
4. There are many important practical applications of modular arithmetic that are best understood by viewing the modular arithmetic in a group theory framework. Examples include the check digits on UPC codes on retail items, ISBN numbers on books, and credit card numbers. In many cases the check digit is the inverse of a weighted sum modulo an integer (10 in the case of a UPC number, 11 in the case of an ISBN number, 9 in the case of Visa travelers checks).
5. Many games can be understood by viewing them as permutation groups. Two examples are the 15 puzzle and the Rubik cube.
6. High school math teachers should be adept at looking at data and making plausible conjectures and generalizations. They should also teach their students to do this. This is a skill that can be learned with practice. Groups and rings provide abundant opportunities for developing this skill.
7. Many people are not comfortable with abstract concepts nor adept at abstract reasoning. The ability to think abstractly is a valuable asset. Abstract algebra helps develop this ability.
8. Abstract Algebra is an ideal capstone course for math ed majors and for those who will go on to grad school in math. Throughout the course they review things like 1-1 functions, onto functions (surprisingly few senior math ed majors understand these ideas well); equivalence relations; basic concepts from linear algebra such as how to multiply matrices, properties of determinants, how to compute a determinant, how to compute the inverse of a matrix, how to tell if a matrix has an inverse, linear transformations (which are group homomorphisms); properties of complex numbers; properties of integers (Euclid's lemma, division algorithm, criterion for divisibility by 9 or 11 or 4); math induction (another important topic that many students do not seem to understand well when they begin an abstract algebra course–this is especially the case for statements that do not involve sums of series); and properties of polynomials (division algorithm, remainder theorem, factor theorem, number of zeros is at most the degree, unique factorization).
9. Doing well in an abstract algebra course is a confidence builder and sometimes causes students to think about going on the graduate school. I once had a student who did extremely well in abstract algebra who went to medical school and now has a high position in the Center for Disease Control in Atlanta. About 20 years after she took the course I met her for dinner while I was at a meeting in Atlanta. I jokingly said to her "Did you use any abstract algebra in med school?" She immediately responded by saying "Abstract algebra was very valuable to me in med school." I asked how. She said that whenever she was taking a difficult course she said to herself "If I can get an A in abstract algebra I can get an A is any course." She was perfectly serious. Many people who start out intending to be high school math teachers or even teach high school math for several years decide to go to grad school in math for an advanced degree (I and many others loved the course and wanted to go to grad school to continue studying the subject). Taking abstract algebra and doing well makes such a move more likely and easier to do.

- Joseph Gallian