MATH 413
Fall 2009
Lab 1 - Week of September 7

Programs used: `fixed point`, `bisection`

This assignment will compare the Bisection Method and the Fixed Point Method in finding the roots of polynomials. In particular, the goal is to find the points of intersection of the two curves:

$$f_1(x) = x^2 + 2x + 3 \qquad \text{and} \qquad f_2(x) = -x + 2$$

In all cases use $\varepsilon = 0.000001$.

- Graph the two curves to see where they intersect. For each point of the intersection α, choose an interval (x_{upper}, x_{lower}) and use the bisection method to approximate each α. (3 pts)

- Find an appropriate fixed point function g and use the values x_{upper} and x_{lower} to approximate each α. Should both x_{upper} and x_{lower} fail to find the root, find a different starting value. (3 pts)

- Use Maple's solving command to find the actual values of α. Compare bisection and the fixed point methods with regards to efficiency and accuracy. (4 pts)

MATH 413
Fall 2009
Lab 2 - Week of September 14

Programs used: `M modification`, `newtons method`

The first part of this assignment will compare the M modification of the fixed point method to the methods used in Lab 1. In particular, the goal is to find the points of intersection of the two curves:

$$f_1(x) = x^2 + 2x + 3 \qquad \text{and} \qquad f_2(x) = -x + 2$$

Use $\varepsilon = 0.000001$.
Recall from Lab 1 that the actual points of intersection are $\alpha_1 = -2.618034$ and $\alpha_2 = -0.381966$.

- Choose an initial guess x_0 and an appropriate value M to use M modification to approximate each value α. (2 pts)

- Compare the three methods you've used to find these points of intersection (bisection, fixed point, and M modification) with regards to efficiency, accuracy, and ease of use. (2 pts)

The second part of the assignment will explore Newton's method. We consider the function $f(x) = \exp(x-2) - x + 1$.

- Plot the graph of f. (2 pts)

- Verify that $x = 2$ is a root of f multiplicity 2. (2 pts)

- Use Newton's method with $\varepsilon = 0.000001$ and $x_0 = 0$ to approximate the root. (2 pts)

MATH 413
Fall 2009
Lab 3 - Week of September 21

Programs used: `P modification`, `secant method`

As a continuation of last week's lab, consider the function $f(x) = \exp(x-2) - x + 1$.

- Use the P-modification program to estimate the roots of the function. Try at least 3 values of P. (3 pts)
- Use the secant method program to estimate the roots of the function. Try at least 3 different starting intervals. (3 pts)
- Compare the three methods you've used to find these points of intersection (Newton's method, P modification, and secant method) with regards to efficiency, accuracy, and ease of use. (4 pts)

MATH 413
Fall 2009
Lab 4 - Week of September 28

Programs used: `secant method, U modification`

We use the function $f(x) = e^(x - 2) - x + 1$ in all examples, and error tolerance $\varepsilon = 0.00001$.

- Use the Secant Method to approximate the root of the function. Try at least 3 intervals, centered around the actual root $x = 2$. (3 pts)
- Use the U modification of Newton's Method to find the root of the function. Choose starting values equal to the radius of the intervals you chose in part (1). (3 pts)
- Compare the methods with regards to efficiency, accuracy and with respect to whatever else you may find worth noticing. (4 pts)

MATH 413
Fall 2009
Lab 5 - Week of October 5

Programs used: `Gaussian Elimination`, `Gaussian Elimination with S`, `Gaussian Elimination with P`, `Gaussian Elimination with P and S`

For the first part of this experiment, consider the system

$$\begin{bmatrix} 10 & 7 & 8 & 7 & 32 \\ 7 & 5 & 6 & 5 & 23 \\ 8 & 6 & 10 & 9 & 33 \\ 7 & 5 & 9 & 10 & 31 \end{bmatrix}$$

- Find an approximate solution to the system using each of the four programs listed above. (2 pts)
- Using these approximations, make a "'guess"' as to the actual solution. Verify that your guess is correct. (1 pts)
- Find the absolute error of the result for each method. Comment on your results. (3 pts)

Next, consider the system

$$\begin{bmatrix} -0.123 & 45.6 & 7.89 & 100 \\ 0 & -0.0123 & 4.56 & 200 \\ 0 & 123 & -5.67 & 300 \end{bmatrix}$$

- Find an approximate solution to the system using each of the four programs listed above. (2 pts)
- Comment on your results. Give at least one reason why a pivoting method will yield considerably different results than a method that does not use pivoting. (2 pts)

MATH 413
Fall 2009
Lab 6 - Week of October 19

Programs used: `Jacobi, Gauss Seidel`

$$(A)\left\{\begin{array}{ccccc} 5x_1 & -x_2 & +2x_3 & -x_4 & =3 \\ x_1 & +4x_2 & -x_3 & +x_4 & =2 \\ x_1 & -x_2 & -4x_3 & +x_4 & =5 \\ & x_2 & & -3x_4 & =0 \end{array}\right\} \text{ and } (B)\left\{\begin{array}{ccccc} 2x_1 & -x_2 & +2x_3 & -x_4 & =0 \\ x_1 & +3x_2 & -x_3 & +x_4 & =2 \\ x_1 & -2x_2 & +x_3 & +x_4 & =0 \\ & x_2 & +x_3 & -2x_4 & =-1 \end{array}\right\}$$

(In all cases, use $\varepsilon = 0.00001$.)

- Check that the exact solution is $(1, 0, -1, 0)$ for both systems.
- Determine whether each system is diagonally dominant. When we solve the systems by Jacobi Iteration with certain initial guess and tolerance $\varepsilon = 0.00001$, can we be sure it will converge to the exact solution? Why or why not?
- Solve each system by Gauss-Seidel with the initial guess $(0, 0, 0, 0)$ first and then $(1, 2, 3, 4)$.
- Solve each system by Jacobi Iteration with the initial guess $(0, 0, 0, 0)$ first and then $(1, 2, 3, 4)$.
- Comment on your results. What problems occur, and why? Why would one method converge faster than another?

MATH 413
Fall 2009
Lab 7 - Week of October 26

Part I

Programs used: `SOR`

$$\text{(A)} \left\{ \begin{array}{rrrrl} 5x_1 & -x_2 & +2x_3 & -x_4 & = 3 \\ x_1 & +4x_2 & -x_3 & +x_4 & = 2 \\ x_1 & -x_2 & -4x_3 & +x_4 & = 5 \\ & x_2 & & -3x_4 & = 0 \end{array} \right\} \text{ and (B)} \left\{ \begin{array}{rrrrl} 2x_1 & -x_2 & +2x_3 & -x_4 & = 0 \\ x_1 & +3x_2 & -x_3 & +x_4 & = 2 \\ x_1 & -2x_2 & +x_3 & +x_4 & = 0 \\ & x_2 & +x_3 & -2x_4 & = -1 \end{array} \right\}$$

- Solve each system by SOR with the initial guess $(0, 0, 0, 0)$ first and then $(1, 2, 3, 4)$. Use $\varepsilon = 0.00001$.
- Comment on your results. Compare to your results from Lab 6.

Part II

Programs used: `gen newton sys`

Consider the system

$$(1) \quad 5x + \tan(y) = 30$$

$$(2) \quad y - \tan(x) = 21$$

- Graph the system. Restrict the plot to $0 \leq x \leq 10$ and $-2 \leq y \leq 2$.
- Try using Generalized Newton's Method to approximate the solution near $(8, -1.25)$. Try three values of ω between 0 and 2, and $\varepsilon = 0.000001$. (Hint: try at least one $\omega < 1$ and one $\omega > 1$.)
- Reorder the equations (2) (1), and try Generalized Newton's Method again using the same initial values, relaxation factors, and error as in part 1.
- Comment on your results. Suggest a reason why reordering the equations improved results with generalized Newton's method. How does the relaxation factor ω affect your results?

MATH 413
Fall 2009
Lab 8 - Week of November 2

Part I: Newton's Method
Programs used: `newton sys`

Consider the system

$$(1) \quad 5x + \tan(y) = 30$$

$$(2) \quad y - \tan(x) = 21$$

- Recall from last week that we attempted to find a solution to this system using generalized Newton's method.
- Try using Newton's method to approximate the solution near $(8, -1.25)$.
- Reorder the equations (2) (1), and try Newton's method again using the same initial values and error bounds as in part 1.
- Comment on your results. Did you find Newton's method or generalized Newton's method more efficient? Which was easier to use? Which required a more refined initial guess, and why?

Part II: Langrange Interpolation
Programs used: `langrange`

1. Use the table provided to find the value of $f(x)$ at $x = 2.8$ by the Lagrange's method of interpolation.

x	.3	1.3	2.3	3.3	4.3	5.3
$f(x)$	.2955202	.9635581	.7457053	$-.1577457$	$-.9161661$	$-.8322674$

- Use the 1st, 2nd, 3rd, 4th and 5th order interpolating polynomial (order data correctly) to interpolate for $f(2.8)$ and to find the absolute error. (Note : the exact value is $\sin(2.8) = .3349882$.)
- Graph the polynomials of order 2, and 5, compare them to $y = \sin(x)$, which is the exact curve.

MATH 413
Fall 2009
Lab 9 - Week of November 9

Programs used: `lagrange, Newton Fore Diff`

1. M. S. Selim and R. C. Seagraves studied the kinetics of elution of copper compounds from ion-exchange resins. The normality of the leaching liquid was the most important factor in determining the diffusivity. Their data were obtained at convenient values of normality; we desire a table of D for integer values of normality.

N	Dx10^6, cm^2/sec	N	Dx10^6, cm^2/sec
0.0521	1.65	0.9863	3.13
0.1028	2.10	1.9739	3.06
0.2036	2.27	2.443	2.92
0.4946	2.76	5.06	2.07

- Using interpolation up to the highest possible order, find interpolation values for $N = 0$ through $N = 5$.
- Compare the results and pick the best interpolation value for $N = 0$ through $N = 5$.
- Which values yields the worst results? What may be the reason for it?

2. Redo problem 1.a and 1.b using Newton's Forward Differences. Try to find the best choice for both the degree of the polynomial and diagonal. Remember to reorder the data points as needed.

MATH 413
Fall 2009
Lab 10 - Week of November 16

Programs used: `Least squares, differentiation`

1. Given the data

x	0	.25	.66666667	.75	1	1.25
$f(x)$	$-.75$	1.80	3.00	2.90	1.07	$-.8$

- Use the Least Squares program to find a second degree polynomial approximation and a fifth degree polynomial approximation over the interval $[0, \frac{5}{4}]$.
- Find an approximate curve of the form $f(x) = a\sin(x) + b\cos(x)$.
- If you had to extrapolate at $x = 2$, which of these curves would you use and why?

Differentiation:

- Consider $f(x) = \ln(x)$. Approximate the derivative at $x = 100$. Use initial $h = 0.5$. For each of the first three columns, find the best approximation (true value is .01). Which gives the best approximation? Does this always occur for each method at the smallest value of h? Why do you think this happens?
- Now consider $f(x) = x\cos(2x)$. Approximate the first and second derivative at $x = 2.5$, using initial $h = 0.5$. Which of the three methods gives the best approximation of the first derivative?

MATH 413
Fall 2009
Lab 11 - Week of November 30

Programs used: `Trapazoidal, Simpsons`

Consider the integral $\int_3^4 (\frac{3}{2}\sqrt{x} + 3x^2)dx$.

- Find the actual value of the integral.
- Use the Trapezoid Rule to evaluate the integral using $n = 2, 4, 8, 20$ subdivisions. Find the absolute error for each approximation.
- Use Simpsons Rule to evaluate the integral using $n = 2, 4, 8, 20$ subdivisions. Find the absolute error for each approximation.
- Compare your results for each method.

Recall from calculus that the proper method of evaluating the integral $\int_a^\infty f(x)dx$ is to evaluate $\lim_{b\to\infty} \int_a^b f(x)dx$. A numerical approach is to make the substitution $t = \frac{1}{x}$, so $dx = -\frac{1}{t^2}dt$, and evaluate $\int_0^{\frac{1}{a}} \frac{1}{t^2} f(\frac{1}{t})dt$. In particular, we note that

$$\int_0^\infty xe^{-x}dx = \int_0^1 xe^{-x}dx + \int_0^1 \frac{e^{-\frac{1}{x}}}{x^3}dx$$

- Find the actual value of $\int_0^\infty xe^{-x}dx$, as well as the integral $\int_0^1 xe^{-x}dx$.
- Use Simpson's Rule with $n = 40$ to approximate $\int_0^1 \frac{e^{-\frac{1}{x}}}{x^3}dx$ by choosing various lower bounds a for the integral $\int_a^1 \frac{e^{-\frac{1}{x}}}{x^3}dx$.
- Approximate the integral $\int_0^\infty xe^{-x}dx$ using your results. Find the absolute error for several different values of a.
- Which value of a yielded the best results, and why?

MATH 413
Fall 2009
Lab 12 - Week of December 7

Programs used: `Gauss Quad Laguerre`

Recall that $\int_0^\infty xe^{-x}dx = 1$.

- Use Gaussian - Laguerre quadrature to approximate the integral with $2, 3, 4$ points.
- Find the absolute error for each approximation. Compare your answers using each method and with Simpson's method from the last lab.